

# THE NATURE OF THERMODYNAMIC ENTROPY

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Thermodynamic entropy is a pathway for transitioning from the mechanical world of fundamental physics theory to the world of probabilities, statistics, microstates, and information theory as applied to analyses of the universe. This paper proposes to explain the physical meaning of thermodynamic entropy, and, its connection to Boltzmann's entropy. The inclusion of Boltzmann's constant in his definition of entropy establishes the connection. The physical basis of Boltzmann's constant and his entropy are explained. These explanations first require new analyses of fundamental properties such as force and mass.

## 1 Introduction

The information contained here is original except for some basics of thermodynamics. The presentation has its theoretical basis originating in new theory incorporating fundamental unity from its start. This paper is not about presenting that theory. However, the premises are the same. The first premise is: There is a single cause for all effects. Theoretical multiple causes are reduced down to one. The second premise is: All properties inferred from empirical evidence must ultimately be expressible in the terms of that empirical evidence. The empirical evidence is always about patterns in changes of velocity. Historically, mass was arbitrarily chosen to be an indefinable property joining the naturally indefinable properties of space and time. This paper begins its analysis by reversing this practice. Mass is defined in terms of distance and duration only. The units of mass are formed from combinations of meters and seconds. Duration is cyclic activity used as a standard for mechanically measuring the passage of time.

Some introductory texts do not make clear that there is a difference between definable properties and indefinable properties. Yet, the problem is there for alert readers to spot. For example, distance and duration are defined in terms of natural properties such as atomic frequency and the distance that light travels. However, the definition of the unit of measurement of mass is a human constructed object.

Older texts used to make clear why the unit of mass is not definable in terms of a natural property. The reason is that mass is not a naturally indefinable property. An indefinable property is one that cannot be defined in terms of other pre-existing properties. Neither distance nor duration can be defined in terms of pre-existing properties. Yet they are the properties from which all empirical evidence is gathered. Therefore, the definitions of their units can be set as consistent measures of natural properties.

### 1.1 New Definitions for Mass and Force

The practice of distinguishing between definable and indefinable properties is here expanded upon. An example of the difference between definable and indefinable properties is: Mass is given units of kilograms. Kilograms are not definable in terms of meters and seconds. Therefore, mass was arbitrarily

made into an indefinable property joining with the special status of distance and duration. After this arbitrary act is performed, force does become definable in terms of so-called pre-existing properties. Force is defined in terms of distance, duration and mass. Its units of newtons are defined in terms of meters, seconds and kilograms.

In addition to the naturally indefinable properties of distance and duration the patterns in empirical evidence make clear that there are at least two other properties, force and resistance to force. Those two properties are inferred from empirical evidence and must, therefore, be definable in terms of that empirical evidence. This means that both force and mass must ultimately be expressible in combinations of measures of distance and duration. The units of kilograms are unnecessary and detrimental to theory that makes use of the indefinable status assigned to mass. Fundamental unity is abandoned. When mass is defined in terms of distance and duration, force will also be definable in terms of distance and duration.

The equation  $f/m=a$  shows that if force and mass are expressed in terms of distance and duration only, their units must reduce to those of acceleration. There are a few choices to try, however the one that works is for mass to have the units of inverse acceleration. The units of force would then be those of acceleration divided by those of acceleration. It would be a ratio of like properties. The empirical units, consisting of meters and seconds, cancel and leave force without units. Force will still have a magnitude and units of newtons can be retained for convenience of communication.

This inverse acceleration definition for mass must refer to a real fundamental property. A premise of this paper is that there is only one physical origin for all effects. The acceleration in the denominator of mass is taken to be representative of this single universal cause for all effects. The nature of that single cause is known, but, is not the subject of this paper. However, the concept of mass as inverse acceleration will be tested here. The point demonstrated first is that force without units becomes much more flexible for developing fundamentally unified theory. This flexibility is demonstrated by the example of calculating the universal gravitation constant.

## 2 Universal Gravitational Constant

Newton's formula for the force of gravity contains the universal gravitational constant  $G$ . The question answered here is: What physical phenomenon does  $G$  represent? The answer can be gained from a close inspection of the force formula. Newton's formula for the force of gravity contains expressions for two separate masses:

$$f = \frac{Gm_1m_2}{r^2}$$

However, his fundamental force formula contains an expression that has only one term for mass:

$$f = ma$$

There is an important clue in this difference between the formulas. There is awkwardness between these two formulas that should not persist. This point can be demonstrated by altering the second formula:

$$f = ma = \frac{dE}{dx} = \frac{mv_p dv_p}{dx}$$

The point is that Newton's force formula for gravity should be easily manipulated into all of the forms shown above. What is specifically not clear is how to show that:

$$\frac{Gm_1m_2}{r^2} = \frac{mv_p dv_p}{dx}$$

What I will show is that one is theoretically derivable from the other. Two new clues are available to help decipher this riddle of the force of gravity. One clue is that the normal units for the universal gravitational constant converted to empirical units are:

$$G_{units} = \frac{\text{newtons} \cdot \text{meters}^2}{\text{kilograms}^2} = \frac{\text{meters}^2}{\frac{\text{seconds}^4}{\text{meters}^2}} = \frac{\text{meters}^4}{\text{seconds}^4}$$

Newtons represent unity. Kilograms are replaced by the units of inverse acceleration. The units of  $G$  are velocity to the 4th power. The second clue is that force is dimensionless. The new theoretical tool which this makes available is that a single force can be the product or quotient of any number of other forces. The units still match because there are no units.

The new units of the universal gravitational constant are those of velocity to the fourth power. Inserting these new units into the force of gravity formula allows one to move easily between the different fundamental forms of a force equation. First it is recognized that an incremental change in velocity has units of velocity. Incremental values of change of a property will be preceded by the symbol  $\Delta$ . Units of velocity to the 4th power allow me to anticipate that:

$$G = v_1 \Delta v_1 v_2 \Delta v_2$$

I can then write:

$$f = \frac{Gm_1m_2}{r^2} = \frac{v_1 \Delta v_1 v_2 \Delta v_2 m_1 m_2}{r^2} = \left( \frac{m_1 v_1 \Delta v_1}{r} \right) \left( \frac{m_2 v_2 \Delta v_2}{r} \right)$$

A simple model of the hydrogen atom will be used as a basis for representing a fundamentally important incremental measure of distance. I represent the radius of the atom as  $\Delta x_c$  and the  $r$  in the above equation becomes represented by multiples of the hydrogen radius:

$$r = n_r \Delta x_c$$

The subscript  $c$  appears in the theoretical work from which parts of this paper were drawn. It is most often used there to represent properties of photons. I retain it for consistency with that work, but, will not be speaking here about photons. Here the subscript  $c$  refers specifically to dimensions of the hydrogen atom:

$$f = \frac{1}{n_r^2} \left( \frac{m_1 v_1 \Delta v_1}{\Delta x_c} \right) \left( \frac{m_2 v_2 \Delta v_2}{\Delta x_c} \right) = \frac{1}{n_r^2} \left( \frac{\Delta E_1}{\Delta x_c} \right) \left( \frac{\Delta E_2}{\Delta x_c} \right) = \frac{1}{n_r^2} f_1 f_2$$

When  $n$  is equal to one:

$$f = f_1 f_2$$

This suggests that Newton's basic formula for the force of gravity consists of the product of two other measures of force. I will shortly give a physical interpretation for these two forces. For now I develop further mathematical representation for the force of gravity. Acceleration can be expressed as:

$$a = v \frac{\Delta v}{\Delta x}$$

I substitute acceleration into the force formula given three steps above:

$$f = \frac{1}{n_r^2} m_1 a_1 m_2 a_2$$

Comparing this to Newton's force of gravity formula, I convert the formula above into:

$$f = (a_1 a_2) \frac{m_1 m_2}{n_r^2} = (a_1 a_2 \Delta x_c^2) \frac{m_1 m_2}{n_r^2 \Delta x_c^2} = (a_1 a_2 \Delta x_c^2) \frac{m_1 m_2}{r^2}$$

I conclude that:

$$G = a_1 a_2 \Delta x_c^2$$

What is this acceleration that helps to form the value of  $G$ ? I can reasonably anticipate that our macroscopic concept of gravity is formed from quantum values of a primary value of gravitational force. What I mean is: Two particles of matter, neutrons, would experience a fundamentally important measure of gravity at a distance of the radius of the hydrogen atom.

In order to test this hypothesis I solve for the acceleration, as a remote stationary observer, contained in  $G$ . Since I am thinking in terms of two identical particles, their accelerations are equal and I can write:

$$a_G = \left( \frac{G}{\Delta x_c^2} \right)^{\frac{1}{2}} = \left[ \frac{6.67 \times 10^{-11} \frac{\text{meters}^4}{\text{sec}^4}}{(5.0 \times 10^{-11} \text{meters})^2} \right]^{\frac{1}{2}} = 1.6 \times 10^5 \frac{\text{meters}}{\text{sec}^2}$$

I compare this result with the acceleration predicted by using Newton's gravitational force formula:

$$f = \frac{G m_1 m_2}{r^2} = \frac{\left( 6.67 \times 10^{-11} \frac{\text{meters}^4}{\text{sec}^4} \right) (1.67 \times 10^{-27} \text{kgm})^2}{(5.0 \times 10^{-11} \text{meters})^2} = 7.4 \times 10^{-44} \text{newtons}$$

The acceleration of one of the neutrons is:

$$a = \frac{f}{m_p} = \frac{7.4 \times 10^{-44} \text{newtons}}{1.67 \times 10^{-27} \text{kgm}} = 4.5 \times 10^{-17} \frac{\text{meters}}{\text{sec}^2}$$

Comparing this result to the acceleration obtained using  $G$ , it appears there is no connection. However, there is a connection if I make a comparison using force instead of acceleration. The force on a neutron using the acceleration I obtained from  $G$  is:

$$f = m_p a_G = (1.67 \times 10^{-27} \text{kgm}) \left( 1.6 \times 10^5 \frac{\text{meters}}{\text{sec}^2} \right) = 2.7 \times 10^{-22} \text{newtons}$$

This magnitude of force is the square root of the magnitude of force obtained using Newton's formula. The units of Newton's formula make this into a real dilemma. However, recognizing that in this example force has no units removes this dilemma. One force can be the square root of another force. What can be the physical interpretation of the product of two forces?

The answer to this question comes from recognizing that there are two ways to measure the acceleration of the two neutrons. A remote stationary observer would measure each neutron as having equal accelerations with respect to the observer. A local observer located at one of the neutrons would measure an acceleration of one neutron with respect to the other neutron. The local observer's measure of acceleration would be different from that obtained by the stationary observer.

The path of the theoretical connection between the two is to use force. For the stationary observer there is a different value of force with which to work. He uses a value of force that will predict a neutron's acceleration with respect to him. This is not the same value of force that must be used to measure the acceleration of one proton with respect to the other proton.

For the local observer located at one of the neutrons:

$$f_L = m_p a_L$$

For the remote, stationary, observer both neutrons undergo the same magnitudes of acceleration:

$$f_R = m_p a_R$$

I now use as a guide the formula derived above which shows Newton's force of gravity to be the product of two other forces. In other words, I assume the value of the remotely measured force to be the square of the locally measured force. The mathematical expression of this is:

$$f_R = f_L^2$$

Or:

$$m_p a_R = m_p^2 a_L^2$$

Newton's formula gave me the remote value of acceleration. I solve for the local measure of acceleration:

$$a_L = \left( \frac{m_p a_R}{m_p^2} \right)^{\frac{1}{2}} = \left( \frac{a_R}{m_p} \right)^{\frac{1}{2}} = \left( \frac{4.5 \times 10^{-17} \frac{\text{meters}}{\text{sec}^2}}{1.67 \times 10^{-27} \text{kgm}} \right)^{\frac{1}{2}} = 1.6 \times 10^5 \frac{\text{meters}}{\text{sec}^2}$$

This is the measure of acceleration obtained using  $G$ . I conclude that the mathematical expression of  $G$  is:

$$G = a_{pL}^2 \Delta x_c^2$$

The interpretation for this result is: The fundamental gravitational constant is the square of the local acceleration due to gravity, for a local observer, of one neutron toward another neutron multiplied by the square of the distance between them. The distance is the length of the radius of the hydrogen atom.

There is an appearance of an artificial aspect of this result. Since the acceleration due to gravity is formally defined using the fundamental gravitational constant, then the equation can seem to be defining  $G$  with an expression that ultimately contains  $G$ . This is not the case. The reason is that the acceleration due to gravity in this theory is due to the variation of the one fundamental cause.

That cause is not explained here. Any attempt would be too lengthy. It is simply inserted into this paper as a given. The properties of that cause are responsible for causing all effects. Therefore, the phrase, the local acceleration due to gravity, is to be understood as an aspect of that cause. When read this way, the equation actually defines  $G$  in terms of the fundamental cause.

### 3 Electric Charge and Time

A previous conclusion presented in this paper is that the single original cause is represented by the acceleration that appears in the denominator of mass. Electric charge is a universal constant and is not representative of a property that undergoes acceleration. In the theory from which this work comes, each theoretical cause is representative of a particular aspect of the single original cause. There is also the

premise that all properties inferred from empirical evidence must be expressible in terms of distance and duration. It must be determined how to represent electric charge in terms of distance and/or duration. Its units must be defined in terms of meters and/or seconds.

Electric charge is a very important fundamental constant and the speed of light is a very important fundamental property. The units for the speed of light are meters/sec. There is an anomalous relationship between electric charge and the speed of light:

$$eC = 4.8x10^{-11} ?$$

This magnitude is the size of the radius of the hydrogen atom. Therefore, I take advantage of the anomaly and test new units for electric charge. I use the hydrogen atom to test the concept that electric charge should have empirical units of seconds. I will use  $\Delta x_c$  to represent the radius of the hydrogen atom:

$$\Delta x_c = eC = 4.8x10^{-11} \text{meters}$$

Making this kind of change to the definition of electric charge cannot be rigorously defended in this paper. The manner in which electromagnetic effects are accounted for is not explained here. The goal of this paper is to explain thermodynamic entropy. The results are considered to be convincing.

I let electric charge represent a universally constant measure of duration. It becomes convenient to use the word 'time' to refer to duration. Therefore, instead of electron charge being represented as:

$$e = 1.602x10^{-19} \text{ coulombs}$$

It will be represented as a period of time by:

$$e = 1.602x10^{-19} \text{ seconds}$$

I use the value of electric charge from the *mks* system of units. I found that systems of units that define electric charge using Coulomb's equation setting the proportionality constant to unity give a corrupted value. In the *mks* system, electric charge is defined by referencing it to measures of force and distance, two true empirical properties. The proportionality constant must be determined by experiment. The meaning of Coulomb's equation is revealed through that constant. That meaning is not included here.

## 4 Thermodynamic Entropy

If the premises set for this paper are accepted, at least out of interest, then a clear physical meaning can be offered for thermodynamic entropy. The explanation begins with Planck's constant.

### 4.1 Planck's Constant

Planck's constant is the proportionality constant relating energy to frequency:

$$E = h\omega$$

Force times distance yields units of energy as meters. The units of frequency remain inverse seconds. Therefore, the units of Planck's constant are meters times seconds.

### 4.2 Boltzmann's Constant

Planck's relation between energy and frequency is one of three such relations. There is an analogous relationship between force and frequency. To show this I begin with:

$$h = \frac{E}{\omega}$$

For the hydrogen atom, the first energy level can be represented as an incremental value:

$$E_c = \Delta E$$

So, I write Planck's equation as:

$$h = \frac{\Delta E}{\omega}$$

Dividing both sides by the incremental  $\Delta x_c$ :

$$\frac{h}{\Delta x_c} = \frac{1}{\omega} \frac{\Delta E}{\Delta x_c}$$

Since:

$$f = \frac{\Delta E}{\Delta x_c}$$

Then:

$$\frac{h}{\Delta x_c} = \frac{f}{\omega}$$

Solving for force:

$$f = \frac{h}{\Delta x_c} \omega$$

To divide Planck's constant  $h$  by the radius of the hydrogen atom, I make use of the equality:

$$\Delta x_c = v_c \Delta t_c$$

Substituting:

$$f = \frac{h}{\Delta x_c} \omega = \frac{h}{v_c \Delta t_c} \omega$$

In the terms of current modern physics, this equation is analogous to:

$$f = \frac{h}{C e} \omega$$

Where  $C$  is the speed of light and  $e$  is electric charge. Substituting the appropriate values:

$$f = \frac{h}{\Delta x_c} \omega = \frac{(6.625 \times 10^{-34} \text{ meters} \cdot \text{seconds}) \omega}{\left(2.998 \times 10^8 \frac{\text{meters}}{\text{seconds}}\right) (1.602 \times 10^{-19} \text{ seconds})} = (1.38 \times 10^{-23} \text{ seconds}) \omega$$

I used the empirical units of this paper. Even so, the value of the constant of proportionality is very recognizable. Its magnitude is the same as that of Boltzmann's constant. I offer the result that:

$$f = k\omega$$

Where  $k$  represents Boltzmann's constant. I did not include my usual subscripts because I want to show it in a form consistent with Planck's well-known energy as a function of frequency equation given before it.

### 4.3 Momentum and Frequency

There can also be shown a relationship between photon momentum and frequency. I will solve for the proportionality constant of this relationship. Force is defined as:

$$f = \frac{\Delta P}{\Delta t_c}$$

Then, using the equation derived two steps above, I write:

$$k = \frac{f}{\omega} = \frac{1}{\omega} \frac{\Delta P}{\Delta t_c}$$

Where  $k$  is Boltzmann's constant. Solving for momentum:

$$\Delta P = k\Delta t_c \omega$$

The proportionality constant is:

$$k\Delta t_c = (1.38 \times 10^{-23} \text{ seconds})(1.602 \times 10^{-19} \text{ seconds}) = 2.21 \times 10^{-42} \text{ seconds}^2$$

Introducing a symbol  $b$  for this constant:

$$b = k\Delta t_c = \frac{h}{\Delta x_c} \Delta t_c = \frac{h}{v_c} = 2.21 \times 10^{-42} \text{ seconds}^2$$

The relationship can then be generalized as:

$$P = b\omega$$

### 4.4 Temperature and Frequency

It is of theoretical importance that Boltzmann's constant appears to be a part of the frequency relationships discussed above. This occurs because the preceding sections demonstrated that:

$$h = k\Delta x_c$$

This relationship allows me to mix formulas that contain either of these constants. For example, I can investigate the possible theoretical meaning of equating:

$$E = \frac{3}{2}kT$$

With:

$$E = h\omega$$

Combining these equations yields:

$$\frac{3}{2}kT = h\omega$$

Rearranging terms:

$$\frac{T}{\omega} = \frac{2h}{3k}$$

Substituting:

$$\frac{T}{\omega} = \frac{2}{3}\Delta x_c$$

Solving for  $T$  and substituting for  $\omega$ :

$$T = \frac{2}{3}\Delta x_c \omega = \frac{2}{3}\Delta x_c \frac{E}{h} = \frac{2}{3} \frac{E}{h/\Delta x_c} = \frac{2E}{3k}$$

Energy is given empirical units of meters. Planck's constant then has units of meters times seconds. The units of temperature must be those of meters divided by seconds. The use of meters in temperature represents the units of energy. Temperature is the rate of propagation of kinetic energy between gas atoms. Substituting the magnitude of the radius of the hydrogen atom:

$$T = \frac{2}{3}(4.8 \times 10^{-11} \text{meters}) \frac{E}{h} = \frac{2}{3}(4.8 \times 10^{-11} \text{meters})\omega$$

Yielding the relationship:

$$T = (3.2 \times 10^{-11} \text{meters})\omega$$

Which says, already commonly known, that temperature is directly related to frequency.

## 4.5 The Physical Meaning of Thermodynamic Entropy

Temperature is an integral part of the definition of thermodynamic entropy and must be clearly, correctly, and fully defined. The typical explanation offered that temperature is a measure of average molecular kinetic energy does not fulfill these requirements. A new interpretation was presented above. Temperature is a measure of the rate of exchange of kinetic energy per molecule. Using this interpretation, the following is the explanation of thermodynamic entropy.

Thermodynamic entropy was discovered long before microstates were discovered. Boltzmann's later definition of entropy, using 'phase space cells' and then microstates, included Boltzmann's constant from ideal gas theory, a classically derived constant with units of joules/Kelvin. These units become, in this paper, the empirical units of meters divided by meters per second, reducing down to seconds.

Clausius' definition of thermodynamic entropy is precise. It is defined under ideal conditions. It is not a process of molecules rearranging themselves at non-equilibrium rates. It does not include any system for which any part can vary from its average temperature. It does not refer to a general process of achieving thermal equilibrium. There are literally no conditions of disequilibrium. We very closely approximate it by restricting the analysis to infinitesimal changes.

Entropy is defined as a mathematical function representing an ideal relationship of the transfer of energy into or out of a system at constant temperature. Energy in transit is called heat  $Q$ . Thermodynamic entropy's definition is:

$$\Delta S = \frac{\Delta Q}{T}$$

Where  $\Delta S$  is a change in entropy,  $\Delta Q$  is the transfer of energy either into or out of a system, and  $T$  is the temperature of the system in degrees Kelvin. This equation is based upon a Carnot engine.

In the differential form, the equations for a series of Carnot engines accurately represent a continuous path on a generalized work diagram so long as the series of engines represented are quasi-static, i.e. no dissipative effects, and reversible, i.e. returns to initial conditions at the end of each cycle. The differential forms of these equations may be solved by means of calculus for the changes in entropy.

The classical definition of entropy, expressed in terms of macroscopic properties, shows how entropy is calculated, but does not make clear what entropy is. It is a mathematical expression and not an explained physics property. It is temperature that masks the identity of entropy. Temperature is an indefinable property in theoretical physics. It is a fundamentally unique indefinable property along with distance, time, mass, and electric charge. It is assigned indefinable units of measurement called 'degrees'. If the physical empirical action that is temperature was identified then entropy would be explained. The new definition given above for temperature will now be used to explain the physical meaning of thermodynamic entropy.

What is thermodynamic entropy? It is something whose nature should be easily established, because, its derivation is part of the operation of the simple Carnot engine. In fact the answer can be found in the operation of the Carnot engine. The Carnot engine is the most efficient engine, theoretically speaking. Its efficiency is independent of the nature of the working medium, in this case a gas. The efficiency depends only upon the values of the high and low temperatures in degrees Kelvin. Degrees Kelvin must be used because the Kelvin temperature scale is derived based upon the Carnot cycle.

The engine's equation of efficiency and the definition of the Kelvin temperature scale are the basis for the derivation of the equation:

$$\frac{Q_{high}}{T_{high}} = \frac{Q_{low}}{T_{low}}$$

Something very important happens during this derivation that establishes a definite rate of operation of the Carnot cycle. The engine is defined as operating quasi-statically. The general requirement for this to be true is that the engine should operate so slowly that the temperature of the working medium should always measure the same at any point within the medium. This is a condition that must be met for a system to be described as operating infinitesimally close to equilibrium.

There are a number of rates of operation that will satisfy this condition; however, there is one specific rate, above which, the equilibrium will be lost. Any slower rate will work fine. The question is: What is this rate of operation that separates equilibrium from disequilibrium? It is important to know this because it is the rate that becomes fixed into the derivation of the Carnot engine. This occurs because the engine is defined such that the ratio of its heat absorbed to its heat rejected equals the ratio of the temperatures of the high and low heat sources:

$$\frac{Q_{high}}{Q_{low}} = \frac{T_{high}}{T_{low}}$$

Temperature is proportional to the rate of exchange of energy between molecules. It is not quantitatively the same as the rate, because, temperature is assigned arbitrary units of measurement that are not time, distance, or a combination of these two. Temperature is assigned the units of degrees Kelvin and its scale is arbitrarily fitted to the freezing and boiling points of water. The temperature difference between these points on the Kelvin scale is set at 100 degrees.

For this reason, the quantitative measurement of temperature is not the same as the quantitative measurement of exchange of energy between molecules. However, this discrepancy can be moderated with the introduction of a constant of proportionality  $k_T$ :

$$\frac{dQ}{dt} = k_T T$$

The ratio  $dQ/dt$  is the definition of the modified temperature. Multiplying by  $dt$ :

$$dQ = k_T dt T$$

Comparing this equation to the definition of thermodynamic entropy indicates that the differential of entropy appears in the above equation as:

$$dS = k_T dt$$

Both  $dS$  and  $dt$  are variables. It is necessary to determine a value for the constant  $k_T$ . This value may be contained in the ideal gas law:

$$E = n \frac{3}{2} kT$$

Where  $k$  is Boltzmann's constant. If I let  $n=1$ , then the equation gives the kinetic energy of a single molecule. In this case  $E$  becomes  $\Delta E$  an incremental value of energy:

$$\Delta E = \frac{3}{2} kT$$

This suggests that for an ideal gas molecule:

$$\Delta S = \frac{\Delta E}{T} = \frac{3}{2} k$$

In other words, the thermodynamic entropy of a single ideal gas molecule is a constant. Substituting for Boltzmann's constant:

$$\Delta S = \frac{3}{2} \left( 1.38 \times 10^{-23} \frac{\text{joules}}{\text{molecule} \cdot ^\circ\text{Kelvin}} \right) = 2.07 \times 10^{-23} \frac{\text{joules}}{\text{molecule} \cdot ^\circ\text{Kelvin}}$$

I have defined Entropy as:

$$\Delta S = k_T \Delta t$$

Therefore, I write:

$$k_T \Delta t = 2.07 \times 10^{-23} \frac{\text{joules}}{\text{molecule} \cdot ^\circ\text{Kelvin}}$$

If I could establish a value for  $\Delta t$ , then I could calculate  $k_T$ . Since this calculation is assumed to apply to a single gas molecule and is a constant value, I assume that in this special case,  $\Delta t$  is a fundamental increment of time. There is a fundamental increment of time previously introduced in this paper. It is:

$$\Delta t_c = 1.602 \times 10^{-19} \text{ seconds}$$

Substituting this value and solving for  $k_T$ :

$$k_T = \frac{2.07 \times 10^{-23} \frac{\text{joules}}{\text{molecule} \cdot \text{Kelvin}}}{1.602 \times 10^{-19} \text{ second}} = 1.292 \times 10^{-4} \frac{\text{joules}}{\text{molecule} \cdot \text{second} \cdot \text{Kelvin}}$$

Substituting empirical units, previously presented, for each quantity and dropping the molecule indicator:

$$k_T = 1.292 \times 10^{-4}$$

The value  $k_T$  is a unit free constant of proportionality. It follows that Boltzmann's constant is defined as:

$$k = \frac{2}{3} \Delta S$$

For the ideal gas equation, the thermodynamic entropy of each molecule is a constant:

$$\Delta S = k_T \Delta t_c$$

However, thermodynamic entropy is a property of an aggregate of molecules. As pressure increases, molecular interactions also increase. I have a value for the constant  $\Delta t_c$  but, the increment of time, as pressure increases, is not a constant. There are a great number of molecules involved and their interactions combine together. The increment of time becomes a variable. I expand the meaning of entropy into a more general form and substitute  $k_T$  into the general thermodynamic definition of entropy:

$$\Delta S = k_T \Delta t$$

The  $\Delta t$  in this equation is not the same as the  $\Delta t_c$  in the equation for a single molecule. In the macroscopic version, it is the time required for a quantity of energy, in the form of heat, to be transferred at the rate represented by the temperature in degrees Kelvin. Substituting this equation for entropy into the general energy equation:

$$\Delta E = \Delta S T = k_T \Delta t T$$

Recognizing that the increment of energy represents an increment of heat  $\Delta Q$  entering or leaving the engine, and solving for  $\Delta S$ :

$$\Delta S = \frac{\Delta E}{T} = \frac{\Delta Q}{T} = k_T \Delta t$$

Solving for  $\Delta t$ :

$$\Delta t = \frac{\Delta S}{k_T} = \frac{\Delta Q}{k_T T}$$

This function of  $\Delta t$  is what would have become defined as the function of entropy if temperature had been defined directly as the rate of transfer of energy between molecules. The arbitrary definition of temperature makes it necessary to include the proportionality constant  $k_T$  in a definition of modified thermodynamic entropy  $\Delta S_m$ . Writing an equation to show this:

$$\Delta S_m = \frac{\Delta Q}{k_T T} = \frac{\Delta Q}{\Delta Q / \Delta t}$$

In particular, for an ideal gas receiving energy from a high temperature reservoir:

$$\Delta S_m = \frac{\Delta Q_{high}}{k_T T_{high}} = \frac{\Delta Q_{high}}{\frac{\Delta Q_{high}}{\Delta t}}$$

For a Carnot engine:

$$\Delta S_m = \frac{\Delta Q_{high}}{k_T T_{high}} = \frac{\Delta Q_{low}}{k_T T_{low}}$$

Therefore:

$$\Delta S_m = \frac{\Delta Q_{high}}{\frac{\Delta Q_{high}}{\Delta t}} = \frac{\Delta Q_{low}}{\Delta t}$$

And the increments of time must be equivalent. This is why the increase in entropy is exactly the opposite of the decrease in entropy for the Carnot engine. The increment of heat entering the engine carries the positive sign, and the increment of energy leaving the engine carries the negative sign.

Temperature is proportional to average kinetic energy because, it is proportional to the rate at which average kinetic energy is transferred between individual molecules. The numerator of modified temperature is average kinetic energy. Modified temperature establishes the point where equilibrium exists. Equilibrium exists when kinetic energy exchanges at the rate of modified temperature or lower.

Now, I consider an engine that operates infinitesimally close to equilibrium conditions, but has heat loss that does not result in work. The heat that is successfully converted into work can be represented by a series of Carnot engines. For the series of Carnot engines, the change in entropy per cycle is zero. The lost heat can be treated as if it just passes through the engines unnoticed. The series of engines is an unaffected pathway for the lost heat to travel from the high heat source to the low heat source.

The lost heat becomes energy no longer available for producing work by the series of Carnot engines. The entropies that are affected are those of the high heat source and the low heat source. The entropies are measures of time required for the lost heat to be released by the high heat source and later absorbed by the low heat source. The net change in entropy is:

$$\Delta S = \frac{Q_{lost}}{T_{low}} - \frac{Q_{lost}}{T_{high}} = \Delta S_m k_T$$

The quantity of heat transferred is the same in both cases. The rates at which that heat will be transferred are different. The low temperature represents a slower rate of exchange of heat than for the high temperature. This means it takes longer for the low temperature source to absorb the quantity of lost heat than it does for the high temperature source to emit the heat.

This time difference is the change that occurs and it is proportional to the measure of change of entropy. The high heat source loses entropy because it requires extra time for the lost heat to leave the source. The low heat source gains entropy because it requires extra time to absorb the heat that is simply passing through the engine without being converted into work. This time difference is what is calculated as thermodynamic entropy. Thermodynamic entropy, referred to as an arrow of time, is an arrow of time.

## 5 Boltzmann's Entropy

The reason for defining thermodynamic entropy using an ideal gas is that as the pressure approaches zero, the exchanges of energy between molecules theoretically reduce down to single exchanges, one at a time without delay. That is the ideal representation for an ideal gas. Thermodynamic entropy applies to

that pressure where the exchanges that occur can be ideally represented by each molecule taking its turn to pass on a measure of average kinetic energy equal to the magnitude of the quantity of average kinetic energy in the numerator of the gas' modified temperature. This process can be accurately modeled by considering all of the gas molecules to be lined up and the kinetic energy of one of them is transferred down the line from molecule to molecule until the energy has been transferred to the last molecule. The time required to complete this process is thermodynamic entropy.

Temperature is proportional to the rate transfer of average molecular kinetic energy of a gas. Thermodynamic entropy for an ideal gas, after the temperature is modified as set forward in this paper, is the time required for energy to be introduced into the gas, from external individual molecule or molecules to an individual gas molecule, at a rate set by the modified temperature of the gas. The modified temperature is the rate at which energy is transferred internally or locally between molecules. The time represented by the thermodynamic entropy of an ideal gas is the time required for each individual molecule to transfer its kinetic energy to another molecule, in turn, until all transfers are completed.

The universal gas constant  $R$  is defined as the pressure times volume of a mole of gas divided by the temperature of the triple point of water defined as 273.16 degrees Kelvin. It is usually written as:

$$pV = RT$$

Below, the pressure times volume, according to the kinetic theory of gases, is equal to the universal gas constant times temperature and to  $2/3$  the internal kinetic energy  $U$  of  $N$  molecules of gas:

$$pV = RT = NkT = N \frac{1}{2} m \overline{v^2} = \frac{2}{3} U$$

The average molecular kinetic energy divided by modified temperature is equal to the time period, as defined in this paper, required for the increment of energy to be transferred:

$$\frac{\frac{1}{2} m \overline{v^2}}{k_T T} = \Delta t_c$$

Boltzmann's constant is defined, in this paper, as the first equals term and by thermodynamics as the second equals term:

$$k = \frac{2}{3} k_T \Delta t_c = \frac{R}{N_0}$$

Where  $N_0$  is Avagadro's number. Solving for  $R$ :

$$R = \left(\frac{2}{3}\right) N_0 k_T \Delta t_c$$

Substituting the appropriate values:

$$R = \left(\frac{2}{3}\right) \left(6.02 \times 10^{23} \frac{\text{molecules}}{\text{mole}}\right) (1.292 \times 10^{-4}) (1.602 \times 10^{-19} \text{seconds}) = 8.31 \left(\frac{\text{molecules} \cdot \text{seconds}}{\text{mole}}\right)$$

For one mole of gas and dropping the molecule indicator:

$$R = \left(\frac{2}{3}\right) (6.02 \times 10^{23}) (1.602 \times 10^{-19} \text{seconds}) = 8.31 \text{ seconds}$$

The universal gas  $R$  constant is the total time required for a mole of ideal gas to transfer average internal molecular kinetic energy from single molecule to single molecule without delay between exchanges at the

rate set by its modified temperature until Avogadro's number  $N_0$  of molecules is reached. Boltzmann's constant is the time period represented by the universal gas constant reduced to single molecule status:

$$k = \frac{R}{N_0} = \frac{8.31 \text{ seconds}}{6.02 \times 10^{23}} = 1.38 \times 10^{-23} \text{ seconds}$$

Boltzmann's constant is a measure of time that is proportional to the period of time necessary for a single exchange of average kinetic energy to occur between two molecules of an ideal gas:

$$k = \left(\frac{2}{3}\right) k_T \Delta t_c = (8.61 \times 10^{-5}) \Delta t_c$$

The actual time period is given by:

$$\Delta t_c = \left(\frac{3}{2}\right) \frac{k}{k_T} = (1.602 \times 10^{-19} \text{ seconds})$$

The number of possible arrangements for a mole of ideal gas is infinite. Boltzmann's entropy requires there to be a limited number of possible arrangements. Classically, his entropy assumed that the volume of the mole of gas could be divided into a limited number of cells available to be occupied. In quantum theory, there are a natural limited number of available arrangements. Instead of arbitrary cells, there are a limited number of actual microstates which particles might occupy. If the concept of microstates is idealized so that all microstates are equally likely to be occupied, then, I can write:

$$S = k \Omega$$

This is not the definition of Boltzmann's entropy even though  $\Omega$  is the number of microstates. The inclusion of Boltzmann's constant causes this calculation to be analogous to that of thermodynamic entropy. The number of microstates simulates a number of ideal gas molecules. The entropy calculation simulates the calculation of internal entropy for a mole of ideal gas. The solution is proportional to the time period required for the simulated ideal gas molecules to transfer their individual average kinetic energies from one molecule to the next, without delay, until the number of simulated molecules equals  $\Omega$ .

The calculation of internal thermodynamic entropy for equivalent numbers of ideal gas molecules is the same for all temperatures. The calculation of the entropy represented by the above equation for a number of microstates equal to an equivalent number of ideal gas molecules will yield a solution identical to an analogous calculation for the equal number of ideal gas molecules. Boltzmann's entropy is defined as:

$$S = k \log \Omega$$

Therefore, Boltzmann's entropy is proportional to the time period of a single transfer of ideal gas molecule energy times the logarithm of the number of microstates. The units of seconds carried along by Boltzmann's constant have become irrelevant. Boltzmann's constant can be set to unity without units. Its connection to thermodynamic entropy is already lost. The loss of this connection allows for entropy to represent whatever the theorist deems useful such as statistical entropy and information entropy.

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